# TRIGONOMETRIC EQUATIONS

## DEFINITION

The equations involving trigonometric function of unknown angles are known as Trigonometric equations.

A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.

## PERIODIC FUNCTION

A function f(x) is said to be periodic if there exists T > 0 such that f(x + T) = f(x) for all x in the domain of definitions of f(x). If T is the smallest positive real number such that f(x + T) = f(x) , then it is called the fundamental period (or) period of f(x)

| Function | Period |
|---|---|
| sin (ax + b) , cos (ax +b), sec (ax + b) , cosec (ax +b ) | $2\pi/a$ |
| tan (ax + b), cot (ax +b) | $\pi/a$ |
| \| sin (ax + b), \| cos (ax +b) \| , \| sec (ax +b) \| , \| cosec (ax +b ) \| | $\pi/a$ |
| \| tan (ax + b ) \| , \| cot (ax +b ) \| | $\pi/2a$ |

The period of sinx, cosec x, cos x, sec x is $2\pi$ and period of tan x, cot x, is $\pi$ .

## TRIGONOMETRICAL EQUATIONS WITH THEIR GENERAL SOLUTION

| Trigonometrical equation | General solution |
|---|---|
| If $\sin\theta = 0$ | then $\theta = n\pi$ : $n \in I$ |
| If $\cos\theta = 0$ | then $\theta = (n\pi + \pi/2) = (2n+1)\pi/2$ : $n \in I$ |
| If $\tan\theta = 0$ | then $\theta = n\pi$ : $n \in I$ |
| If $\sin\theta = 1$ | then $\theta = 2n\pi + \pi/2 = (4n+1)\pi/2$ : $n \in I$ |
| If $\cos\theta = 1$ | then $\theta = 2n\pi$ : $n \in I$ |
| If $\sin\theta = \sin\alpha$ | then $\theta = n\pi + (-1)^n \alpha$ where $\alpha \in [-\pi/2, \pi/2]$ : $n \in I$ |
| If $\cos\theta = \cos\alpha$ | then $\theta = 2n\pi \pm \alpha$ where $\alpha \in (0, \pi]$ : $n \in I$ |
| If $\tan\theta = \tan\alpha$ | then $\theta = n\pi + \alpha$ where $\alpha \in (-\pi/2, \pi/2]$ : $n \in I$ |
| If $\sin^2\theta = \sin^2\alpha$ | then $\theta = n\pi \pm \alpha$ : $n \in I$ |
| If $\cos^2\theta = \cos^2\alpha$ | then $\theta = n\pi \pm \alpha$ : $n \in I$ |
| If $\tan^2\theta = \tan^2\alpha$ | then $\theta = n\pi \pm \alpha$ : $n \in I$ |
| If $\left.\begin{matrix}\sin\theta = \sin\alpha \\ \cos\theta = \cos\alpha\end{matrix}\right\|^*$ | then $\theta = 2n\pi + \alpha$ : $n \in I$ |
| If $\left.\begin{matrix}\sin\theta = \sin\alpha \\ \tan\theta = \tan\alpha\end{matrix}\right\|^*$ | then $\theta = 2n\pi + \alpha$ : $n \in I$ |
| If $\left.\begin{matrix}\tan\theta = \tan\alpha \\ \cos\theta = \cos\alpha\end{matrix}\right\|^*$ | then $\theta = 2n\pi + \alpha$ : $n \in I$ |

* Every where in this chapter "n" is taken as an integer.
* If $\alpha$ be the least positive value of $\theta$ which satisfy two given trigonometrical equations , then the general value of $\theta$ will be $2n\pi + \alpha$

## GENERAL SOLUTION OF TRIGONOMETRICAL EQUATION $a\cos\theta + b\sin\theta = C$

To solve the equation a cos $\theta$ + b sin $\theta$ = c, subsitute a = r cos $\phi$ , b = r sin $\phi$ such that

$$r = \sqrt{a^2 + b^2}\ ,\ \phi = \tan^{-1}\frac{b}{a}$$

Substituting these values in the equation we have r cos $\phi$ cos $\theta$ r sin $\phi$ sin $\theta$ = c

$$\cos(\theta - \phi) = \frac{c}{r} \quad \Rightarrow \quad \cos(\theta - \phi) = \frac{c}{\sqrt{a^2 + b^2}}$$

If $|c| > \sqrt{a^2 + b^2}$ , then the equation ;

a cos $\theta$ + b sin $\theta$ = c has no solution

If $|c| \le \sqrt{a^2 + b^2}$ , then take ;

$$\frac{|c|}{\sqrt{a^2 + b^2}} = \cos\alpha \text{ , so that}$$

$$\cos(\theta - \phi) = \cos\alpha$$

$$\Rightarrow \quad (\theta - \phi) = 2n\pi \pm \alpha$$

$$\Rightarrow \quad \theta = 2n\pi \pm \alpha + \phi$$

## SOLUTIONS IN THE CASE OF TWO EQUATIONS ARE GIVEN

Two equations are given and we have to find the values of variable $\theta$ which may satisfy both the given equations, like

cos $\theta$ = cos $\alpha$ and sin $\theta$ = sin $\alpha$

so the common solution is $\theta = 2n\pi + \alpha$ , $n \in I$

Similarly, sin $\theta$ = sin $\alpha$ and tan $\theta$ = tan $\alpha$

so the common solution is , $\theta = 2n\pi + \alpha$ , $n \in I$

Rule : Find the common values of $\theta$ between 0 and $2\pi$ and then add $2\pi n$ to this common value